

VMC Math Tutorials

Welcome to VMC's math tutorials. The purpose of this tutorial is to help students with important mathematical topics that they will need to know for next year. At VMC we have an advanced math program. This means secondary 1 students do secondary 2 math. The whole year is dedicated to the secondary 2 math program but before each secondary 2 topic and chapter, we first cover basic secondary 1 math that you are missing.

The topics that you will cover here are basic but necessary for you to know for next year. You will find several sections about various topics. Each topic, has definitions, explanations and examples to help you understand.

Knowing these basic topics well will really help you next year. Please review this tutorial several times if necessary, until you completely understand and a good idea is to practice these topics using the internet.

Section 1:

Mental Math

Being able to add, subtract, divide and multiply numbers mentally is a very important and useful tool. Students who are strong in mental math have a much easier time with not only basic math skills but with future problem solving.

Here is a multiplication table to help with this process. You should know this table well and practice other mental math skills.

Multiplication Table

x	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Section 2

Factors

A **factor** is a number that divides into another number. Factoring a number means finding all the numbers (factors) that can divide it perfectly.

Ex: 16 --> 1, 2, 4, 8, 16 These are the factors of 16
20 --> 1, 2, 4, 5, 10, 20 These are the factors of 20
45 --> 1, 3, 5, 9, 15, 45 These are the factors of 45

Multiples

A **multiple** of a number is the product of that number and any other whole number. That may sound complicated but in simple terms, a multiple is a number multiplied by 1, by 2, by 3, by 4, by 5...etc.

Ex: 2 --> 2, 4, 6, 8, 10.... 2,4,6,8... are multiples of 2
5 --> 5, 10, 15, 20, 25... 5,10,15...are multiples of 5
7 --> 7, 14, 21, 28, 35... 7,14,21...are multiples of 7

Greatest Common Factor: GCF

The **greatest common factor**, or GCF, is the greatest factor that divides both numbers.

→ **GCF:** Is the largest number that divides both numbers

Ex: 20: ___ 1, 2, 4, 5, **10**, 20 these are factors of 20

30: ___ 1, 2, 3, 5, 6, **10**, 15, 30 these are factors of 30

The GCF of 20 & 30 is **10**.

Lowest Common Multiple: LCM

A **common multiple** is a number that is a multiple of two or more numbers.

The **lowest common multiple** (LCM) of two numbers is the smallest number (not zero) that is a multiple of both.

Ex: 4----4, 8, **12**, 16, 20, 24, 28, 32, 36 the **LCM** of 4 & 6 is **12**

6----6, **12**, 18, 24, 30, 36, 42, 28

Section 3

Fractions

What is a fraction: there are different ways of thinking about it.

1. A fraction represents a part of a whole. A fraction represents one piece of something larger.

Ex: An easy example would be a slice of a pizza. If a pizza was cut into 4 slices and you ate 3 of them, then you ate $\frac{3}{4}$ of the pizza.

2. It also represents a division of two numbers. One way to think of a fraction is as a division that hasn't been done yet.

Ex: $\frac{1}{2}$ means $1 \div 2$ $\frac{4}{5}$ means $4 \div 5$

***A fraction is another way of writing a division of two numbers.

3. It represents a decimal number

Ex: $0.5 \rightarrow \frac{1}{2}$ $1.5 \rightarrow \frac{15}{10}$

Identification of Simple Fractions

All fractions have three parts: a numerator, a denominator, and a division symbol.

Example 1: Find the numerator, denominator, and division symbol for the simple fraction.

$$\frac{3}{4} \quad (\text{means } 3 \text{ divided by } 4)$$

Answer. The numerator is 3, the denominator is 4, and the division symbol is /.

Types of fraction:

Proper fraction: numerator is less than the denominator. $\frac{3}{4}$

Improper fraction: numerator is greater than or equal to denominator. $\frac{5}{4}$

Mixed number: whole number and a fraction. $2\frac{1}{3}$

Equivalent fractions: fractions that represent the same number. $\frac{1}{2} = \frac{3}{6} = \frac{10}{20}$

Reducing fractions

Reducing a fraction (also called simplifying a fraction) simply means making the fraction as small as possible.

Ex: $\frac{50}{100}$ is equal to $\frac{1}{2}$

To reduce a fraction to lowest terms, divide the numerator and denominator by their Greatest Common Factor (GCF). Divide the numerator and denominator by the largest number that they both have in common.

Ex: $\frac{20}{30}$ What number divides into 20 & 30 perfectly. The answer is 10.

Therefore $\frac{20 \div 10}{30 \div 10}$ which equals $\frac{2}{3}$ this fraction is now reduced completely.

If you know the GCF of both the numerator and denominator you can reduce the fraction very easily and quickly. If not you simply keep reducing the fraction until you're done.

Method 1: Using GCF

Ex: $\frac{36}{48} \div 12$ equals $\frac{3}{4}$ This is fast because I used their GCF

Method 2: Using numbers I know divide into both numbers. It could take a few steps.

$\frac{36}{48} \div 2$ equals $\frac{18}{24} \div 6$ equals $\frac{3}{4}$ Now it is reduced.

Adding and Subtracting Fractions

Adding and subtracting fractions with the same denominator is very easy. You simply add or subtract the numerators and keep the same denominator. (Do not + or – the denominators)

$$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$$

Adding and subtracting fractions with the different denominators is more work.

- You must put both fractions on a common denominator before adding or subtracting their numerators. To do this you must figure out the multiples both denominators have in common.

$\frac{1}{6} + \frac{2}{4}$ So what are the common multiples of both 4 and 6. Their LCM, lowest common multiple they have is 12. I know $6 \times 2 = 12$ and $4 \times 3 = 12$.

- We can make equivalent like fractions by multiplying the numerator and denominator of each fraction by the factor(s) needed.

$$\frac{1 \times 2}{6 \times 2} + \frac{2 \times 3}{4 \times 3} \Rightarrow \frac{2}{12} + \frac{6}{12} = \frac{8}{12} \text{ we can reduce this by } \div 4 \text{ giving us } \frac{2}{3} \text{ as final answer}$$

We did not change both fractions but made equivalent fractions to enable us to add them. $1/6$ became $2/12$ while $2/4$ became $6/12$. Remember whatever you do to the denominator you must also do to the numerator or else the fraction is not the same.

***Subtraction works the same way.

Multiplying Fractions

Multiplying fractions is an easy process. You simply have to multiply the numerators together and multiply the denominators together. Then reduce your answer.

$$\text{Ex: } \frac{2}{4} \times \frac{3}{5} = \frac{6}{20} \text{ we can now reduce the fraction. } \frac{6}{20} \div 2 = \frac{3}{10}$$

$$\text{Ex: } 2 \times \frac{3}{7} \rightarrow \text{like saying } \frac{2}{1} \times \frac{3}{7} = \frac{6}{7}$$

Dividing Fractions

Dividing fractions is also quite simple, as it is very much like multiplying fraction only with an extra step.

To divide any number by a fraction:

First step: Find the reciprocal of the second fraction. (Flip the second fraction)

Second step: Multiply the numerators together and multiply the denominators together.

Third step: Simplify the resulting fraction if possible.

Ex: $\frac{2}{5} \div \frac{3}{4} \rightarrow$ becomes $\frac{2}{5} \times \frac{4}{3} = \frac{8}{15}$ Done. Fraction is already reduced.

Ex: $\frac{4}{7} \div \frac{2}{2} \rightarrow$ becomes $\frac{4}{7} \times \frac{1}{2} = \frac{4}{14}$ Done. $\rightarrow \frac{2}{7}$ reduced

Section 4

Numbers and The Number Line

This section will discuss numbers; whole numbers, decimal numbers, and fractions, as well as a number line.

Whole number: A whole number can't be a fraction of a number, a percentage, or have a decimal.

Ex: 1,2,3,4,...35...150..etc

Decimal number:

Our decimal system lets us write numbers as large or as small as we want, by using a decimal point. Digits can be placed to the left and right of a decimal point, to indicate numbers greater than one or less than one.

The decimal point helps us to keep track of where the "ones" place is. It's placed just to the right of the ones place. As we move right from the decimal point, there is the tenths, hundredths etc...

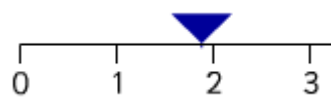
Ex: 1.5 or 2.25 or 4.571 or 120.358 etc...

The Number line:

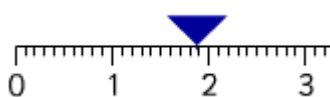
The number line helps us understand whole numbers, decimal numbers and fractions. It visually shows us what they are, how much they are worth and where they belong.

Examples of Number lines:

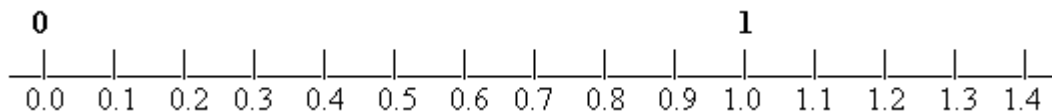
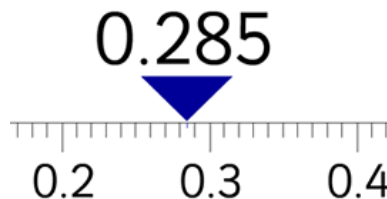
1.



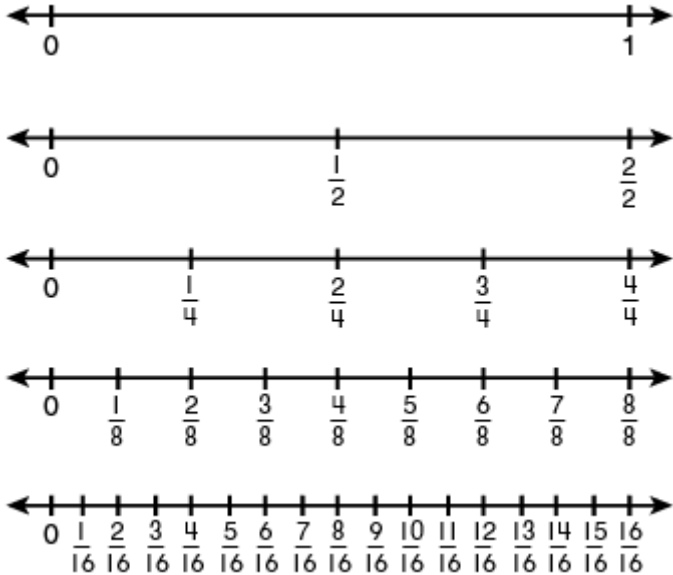
2.



3.



Number line with fractions:



Section 5

Exponents

Exponents are a shorthand way to write a multiplication of the same number many times over. An exponent tells us how and times a number is being multiplied by itself.

Ex: $5 \times 5 \times 5 = 5^3$ The expression " 5^3 " is read as "five to the power of 3"

5^3 -----5 is called the base, the 3 is known as the power of the exponent.

Examples:

$$7 \times 7 \times 7 \times 7 = ? \quad 7^4$$

$$2 \times 2 \times 2 \times 2 \times 2 \times 2 = ? \quad 2^6$$

$$1^{10} = 1$$

Section 6:

Percent

Percent means "out of 100." We can use the percent symbol (%) as a handy way to write a fraction with a common denominator of 100. For example, instead of saying "8 out of every 100 professional basketball players are female," we can say "8% of professional basketball players are female."

A percent can always be written as a decimal, and a decimal can be written as a percent, by moving the decimal point two places to the right like this:

$$0.5 = 50\% \quad 0.25 = 25\% \quad 1.2 = 120\% \quad 0.01 = 1\%$$

We use the percent symbol (%) to express percent. Percents are used everywhere in real life, so you'll need to understand them well. Here are three ways to write the same thing:

$$15\% = 15/100 = 0.15$$

Remember:

- a decimal number = a fraction = a percentage

So this means: $0.5 = \frac{1}{2} = 50\%$ these are all equal just written differently.

Ex: If you receive 12/15 on your math test, what percent did you get on the test?

1. You must do 12 divided by 15 which equals = 0.8
2. You then take 0.8×100 (to get your answer in percentage) and you get = 80%

Therefore you received 80% on your math test.

Section 7

Order of Operations

Order of operation is the order that you must follow to solve or complete a mathematical equation or expression.

BEDMAS is a simple way to remember which steps you should do first, second, third, when you are solving a math problem.

B-brackets

E-exponents

D-division → multiply or divide (what ever comes 1st from left to right)

M-multiplication

A-addition → addition or subtract (what ever comes 1st from left to right)

S-subtraction

Simple examples using Order of Operation

$$\begin{aligned} 6 + 7 \times 8 &= 6 + 7 \times 8 && \text{Multiplication 1}^{\text{st}} \\ &= 6 + 56 && \text{Addition 2}^{\text{nd}} \\ &= 62 \end{aligned}$$

$$\begin{aligned} 16 \div 8 - 2 &= 16 \div 8 - 2 && \text{Division 1}^{\text{st}} \\ &= 2 - 2 && \text{Subtraction 2}^{\text{nd}} \\ &= 0 \end{aligned}$$

$$\begin{aligned} (25 - 11) \times 3 &= (25 - 11) \times 3 && \text{Brackets} \\ &= 14 \times 3 && \text{Multiplication} \\ &= 42 \end{aligned}$$

Example 2: Evaluate $3 + 6 \times (5 + 4) \div 3 - 7$ using the order of operations.

Solution:

$$\begin{aligned} \text{Step 1: } 3 + 6 \times (5 + 4) \div 3 - 7 &= 3 + 6 \times 9 \div 3 - 7 && \text{Brackets} \\ \text{Step 2: } 3 + 6 \times 9 \div 3 - 7 &= 3 + 54 \div 3 - 7 && \text{Multiplication} \\ \text{Step 3: } 3 + 54 \div 3 - 7 &= 3 + 18 - 7 && \text{Division} \\ \text{Step 4: } 3 + 18 - 7 &= 21 - 7 && \text{Addition} \\ \text{Step 5: } 21 - 7 &= 14 && \text{Subtraction} \end{aligned}$$

Example 3: Evaluate $9 - 5 \div (8 - 3) \times 2 + 6$ using the order of operations.

Solution:

$$\begin{aligned} \text{Step 1: } 9 - 5 \div (8 - 3) \times 2 + 6 &= 9 - 5 \div 5 \times 2 + 6 && \text{Brackets} \\ \text{Step 2: } 9 - 5 \div 5 \times 2 + 6 &= 9 - 1 \times 2 + 6 && \text{Division} \\ \text{Step 3: } 9 - 1 \times 2 + 6 &= 9 - 2 + 6 && \text{Multiplication} \end{aligned}$$

Step 4:	$9 - 2 + 6$	$= 7 + 6$	Subtraction
Step 5:	$7 + 6$	$= 13$	Addition

In Examples 2 and 3,

- You will notice that multiplication and division were done from left
- Also, addition and subtraction are done from left to right

Example 4: Evaluate $150 \div (6 + 3 \times 8) - 5$ using the order of operations.

Solution:	Step 1:	$150 \div (6 + 3 \times 8) - 5$	$= 150 \div (6 + 24) - 5$	Multiplication inside Brackets
	Step 2:	$150 \div (6 + 24) - 5$	$= 150 \div 30 - 5$	Addition inside Brackets
	Step 3:	$150 \div 30 - 5$	$= 5 - 5$	Division
	Step 4:	$5 - 5$	$= 0$	Subtraction

Examples with exponents:

Simplify $4 + 3^2$.

To solve this, you must first complete the exponent then add in the 4:

$$\begin{aligned}
 &4 + 3^2 && \text{(exponent 1}^{\text{st}}) \\
 &= 4 + 9 && \text{(addition 2}^{\text{nd}}) \\
 &= 13
 \end{aligned}$$

Simplify $4 + (2 + 1)^2$

$$\begin{aligned}
 &4 + (2 + 1)^2 && \text{brackets 1}^{\text{st}} \\
 &= 4 + (3)^2 && \text{exponent 2}^{\text{nd}} \\
 &= 4 + 9 && \text{addition 3}^{\text{rd}} \\
 &= 13
 \end{aligned}$$

Section 8:

Here are a couple of stencils to help you practice some of the topics that were discussed with the answers..

Stencil 1

1. List the factors of 24?
2. What is the LCM of 6 and 9?
3. What is the LCM of 24 and 30?
4. What is the LCM of 25 and 30?
5. Write $23 \times 23 \times 23 \times 23$ as a power.
6. Express $2 \times 10^5 + 3 \times 10^4 + 8 \times 10^3 + 1 \times 10^2 + 5 \times 10 + 2 \times 1$ in standard form.
7. Express 1.234×10^6 in standard form.
8. Calculate $\sqrt{30}$.
9. Evaluate the expression $(3 + 2.2^2) \div 2 + 5.5^2$. Show your work.
10. What is the GCF of 28 and 126?
11. Calculate.
 $\frac{3}{4} + \frac{1}{8}$
12. Calculate.
 $\frac{5}{8} - \frac{1}{2}$
13. Solve
 $\frac{3}{5} + \frac{2}{7}$
14. Solve
 $\frac{7}{8} - \frac{3}{5}$
15. Solve
 $\frac{1}{2} - \frac{3}{13}$
16. Solve
 $4\frac{1}{2} - 1\frac{3}{4}$

17. Calculate

$$\frac{1}{2} \div \frac{2}{3}$$

18. Calculate

$$\frac{1}{8} \div \frac{3}{4}$$

19. Which of the following is greater product?

a. $\frac{1}{2} \times \frac{1}{8}$

b. $\frac{1}{4} \times \frac{1}{6}$

20. Which of the following is greater quotient?

a. $\frac{1}{2} \div \frac{1}{8}$

b. $\frac{1}{4} \div \frac{1}{6}$

Answer Section

1. $24 : 1,2,3,4,6,8,12,24$

2. $\text{LCM} = 18$

3. $\text{LCM} = 120$

4. $\text{LCM} = 150$

5. 23^4

6. $238\ 152$

7. $1\ 234\ 000$

8. $\sqrt{30} = 5.5$

9.

$$\begin{aligned} & (3 + 2 \cdot 2^2) \div 2 + 5.5^2 \\ &= (3 + 4 \cdot 84) \div 2 + 30.25 \\ &= (7.84) \div 2 + 30.25 \\ &= 3.92 + 30.25 \\ &= 34.17 \end{aligned}$$

10. $\text{GCF} = 14$

11. $\frac{7}{8}$

12. $\frac{1}{8}$

13.

$$\begin{aligned}\frac{3}{5} + \frac{2}{7} \\ &= \frac{21}{35} + \frac{10}{35} \\ &= \frac{31}{35}\end{aligned}$$

14.

$$\begin{aligned}\frac{7}{8} - \frac{3}{5} \\ &= \frac{35}{40} - \frac{24}{40} \\ &= \frac{11}{40}\end{aligned}$$

15.

$$\begin{aligned}\frac{1}{2} - \frac{3}{13} \\ &= \frac{13}{26} - \frac{6}{26} \\ &= \frac{7}{26}\end{aligned}$$

16.

$$\begin{aligned}4\frac{1}{2} - 1\frac{3}{4} \\ &= \frac{9}{2} - \frac{7}{4} \\ &= \frac{18}{4} - \frac{7}{4} \\ &= \frac{11}{4} \\ &= 2\frac{3}{4}\end{aligned}$$

17.

$$\frac{3}{4}$$

18.

$$\frac{1}{6}$$

19. a. has the greater product.

20. a. has the greater quotient.

Stencil 2

- List all its factors 45.
- List all its factors of 49.
- What is the GCF of 12 and 16?
- What is the GCF of 120 and 240?
- What is the LCM of 6 and 9?
- Write 2^6 as a repeated multiplication.
- Write the number 81 as a power in two different ways.
- What is the prime factorization of 30?
- Write $8 \times 8 \times 8 \times 8 \times 8$ as a power.
- Calculate.
$$\frac{5}{6} - \frac{1}{3}$$
- Calculate.
$$\frac{5}{8} - \frac{1}{2}$$
- Solve
$$\frac{3}{5} + \frac{2}{7}$$
- Solve
$$\frac{8}{9} + \frac{5}{6}$$
- Calculate
$$5 - 1\frac{1}{3}$$

15. Calculate

$$\frac{2}{3} \times \frac{1}{4}$$

16. Calculate

$$\frac{1}{5} \div \frac{1}{2}$$

17. Calculate

$$\frac{2}{9} \div \frac{1}{2}$$

Answer Section

1. 1, 3, 5, 9, 15, 45

2. 1, 7, 49

3. GCF = 4

4. GCF = 120

5. LCM = 18

6. $2 \times 2 \times 2 \times 2 \times 2 \times 2$

7.

$$81 = 9^2$$

$$81 = 3^4$$

8. $30 = 2 \times 3 \times 5$

9. 8^5

10.

$$\frac{1}{2}$$

11.

$$\frac{1}{8}$$

12.

$$\begin{aligned}\frac{3}{5} + \frac{2}{7} \\ &= \frac{21}{35} + \frac{10}{35} \\ &= \frac{31}{35}\end{aligned}$$

13.

$$\begin{aligned}\frac{8}{9} + \frac{5}{6} \\ &= \frac{16}{18} + \frac{15}{18} \\ &= \frac{31}{18} \\ &= 1\frac{13}{18}\end{aligned}$$

14.

$$3\frac{2}{3}$$

15.

$$\begin{aligned}\frac{2}{3} \times \frac{1}{4} \\ &= \frac{2}{12} \\ &= \frac{1}{6}\end{aligned}$$

16.

$$\frac{2}{5}$$

17.

$$\begin{aligned}\frac{2}{9} \div \frac{1}{2} \\ &= \frac{2}{9} \times \frac{2}{1} \\ &= \frac{4}{9}\end{aligned}$$

Section 9

Websites:

Here are a few websites that you can visit to help you practice some more and cover the topics that weren't found in the stencils..

1. <http://www.enchantedlearning.com/math/>

This is a great website to review most of the topics that were discussed and other basic topics you also need to know (long division, multiplication, decimals, percentage and more). Take the time to go over the entire website, practice the exercises and review the answers they provide you.

Fractions:

2. <http://lsc.durhamcollege.ca/quizzes/FractionsExercise.htm>

Percentage and decimal numbers:

3. http://www.mathgoodies.com/lessons/vol4/challenge_vol4.html